Exam. Code : 103204
Subject Code : 1107

B.A./B.Sc. 4th Semester

MATHEMATICS

Paper—II

(Solid Geometry)

Time Allowed—2 Hours] [Maximum Marks—50

Note :— There are ***eight*** questions of equal marks. Candidates are required to attempt any ***four*** questions.

1. (a) A cylinder cuts the plane z = 0 in the curve $x^2 + \frac{y^2}{4} = \frac{1}{4}$, and has its axis parallel to 3x = –6z. Find its equation.

   (b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, z = 0.

2. (a) Find the equation of the right circular cylinder whose guiding curve is the circle passing through the points (2, 0, 0), (0, 2, 0) and (0, 0, 2).

   (b) Find the equation of the right circular cylinder whose guiding circle is :

   $x^2 + y^2 + z^2 - 2x + 4y - 6z - 2 = 0, 2x + 3y + 6z = 0.$

3. (a) Prove that the equation $x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0$ represents a cone, find its vertex.

(b) Find the condition that the plane $lx + my + nz = 0$ may touch the cone $2x^2 - 3y^2 + z^2 = 0$ and find the equation of the reciprocal cone.

4. (a) Find the equation of cone whose vertex is at origin and base curve if $f(x, y) = 0$, $z = k$.

(b) Find the angle between the lines of sections of the following planes and cones :

$$3x + y + 5z = 0,\ 6yz - 2zx + 5xy = 0.$$

5. Reduce $x^2 + 3y^2 + 3z^2 - 2yz - 2x - 2y + 6z + 3 = 0$ to standard form and prove that it represents an ellipsoid.

6. (a) Write down the equation of the surface of revolution obtained by rotating the curve $y^2 + 16z^2 = 4$, $x = 0$ about the z-axis.

(b) Find the locus of the chords of the conicoid $ax^2 + by^2 + cz^2 = 1$ which are bisected at the points (x_1, y_1, z_1)

7. Reduce the equation :

$$3x^2 + 7y^2 + 3z^2 + 10yz - 2zx + 10xy + 4x - 12y - 4z + 1 = 0$$

to the standard form and state the nature of the surface represented by it.

8. (a) A tangent plane to ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$ meets the co-ordinates axes in L,M,N. Prove that the centroid of the triangle LMN lies on $\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=9$.

(b) Find the equation of the tangent plane at the point (x_1, y_1, z_1) of the central conicoid $ax^2 + by^2 + cz^2 = 1$.